Breaking the Cocoon of a Millennial Logical Dilemma: Paradigmatic Innovation in Logical Domain Theory and the Ultimate Resolution of Paradoxes

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Abstract: Traditional logic has long been trapped in the paradoxes of substantive implication, incomplete conceptual relations and logical paradoxes. The theory of logical domains achieves a systematic breakthrough of traditional logic by reconstructing implication relations, completing the conceptual system and unifying logical laws. This article unfolds from three aspects, namely the core of the theory, application value and historical significance, to demonstrate its milestone role in the paradigm innovation of logic and provides new ideas for solving the millennial logical dilemma.

Keywords:logic domain theory; substantial implication; conceptual relationship; logical paradox; paradigm innovation

Introduction

Logic, as a core discipline exploring the laws of human thinking, has been constantly evolving on the path of pursuing precision and universality since Aristotle established the formal logic system in the fourth century BCE. From the syllogism of ancient Greece to the symbolic revolution of modern mathematical logic, logic has continuously deepened the depiction of reasoning rules through the iteration of formal tools. However, while this development has promoted the progress of the discipline, it has also gradually exposed profound contradictions that are disconnected from human natural thinking, which are concentratedly manifested in three core predicaments.

First of all, the truth-value paradoxes of material implication have become an insurmountable obstacle for traditional logic. Since Frege simplified “if p then q” to the truth-functional operation of “$\neg p \vee q$”, the field of logic has long been confronted with the paradox that “false propositions implies all propositions”. For instance, the statement “If the moon were made of cheese, then the earth would be square” is judged as a true proposition in the system of substantive implication. This kind of formalized treatment that divorces from semantic relevance reduces logical deduction to a mere game of symbols, seriously deviating from human intuitive understanding of conditional propositions.

Secondly, the incompleteness of the theory of conceptual relation restricts the depth of logical analysis. Traditional logic has only established four categories and six types of relations, namely, sameness, inclusion (true inclusion in, true inclusion), pure intersection (the so-called conceptual intersection relationship usually referred to), and total difference (opposition, contradiction). It is unable to explain the subcontrary relationship (both the subcontrary relation and the pure intersection relation belong to the intersection relation) which widely exists in natural languages, that “it is impossible for both propositions to be false but possible for both to be true (it is impossible for both to be negated but possible for both to hold true)”. For example, in the conceptual domain of “triangle”, the logical connections between the complement of “obtuse triangle” and “right triangle”, namely “non-obtuse triangle” and “non-right triangle”, have always lacked a systematic explanation within the traditional framework. This results in the failure of conceptual operations in complex semantic scenarios.

Finally, the unsolvable predicament of logical paradoxes continuously challenges the self-consistency of the logical system. The self-referential proposition represented by the liar paradox, “This statement is false”, constantly triggers the logical contradiction of “both true and false” through the circular negation of substantive implication. Although scholars such as Russell and Tarski have attempted to avoid paradoxes by methods of hierarchical theory and language hierarchy and other methods, they have never been able to fundamentally eliminate the threat these paradoxes pose to the logical foundation.

The root cause of these predicaments essentially lies in the fact that traditional logic overly pursues formalization while neglecting the semantic essence. Logical rules should originally be the refinement of the laws of human thinking. However, when the semantic connection between the formal system and natural language is severed, logic loses its core value as a cognitive tool. The theory of logical domain precisely addresses this crux of the problem and systematically reconstructs the traditional logical framework through a dual-track model of “semantic anchoring + logical deduction”. Through four major breakthroughs, namely, innovating the implication relationship, completing the conceptual system, unifying logical laws, and resolving logical paradoxes, this theory not only fills the theoretical gap in the history of both Chinese and foreign logic but also promotes logic to return from the “labyrinth of formalization” to the essence of human thinking, opening up a revolutionary new direction for the development of the discipline.

1. The subversive reconstruction of material implication

1.1 The defects of traditional material implication

In the traditional logical system, material implication simplifies the conditional proposition “p→q” (if p, then q) into the truth-value operation form of “¬p∨q” (not p or q). Although this approach demonstrates simplicity in mathematics and symbolic logic, it has given rise to serious logical paradoxes and semantic ruptures.

Firstly, the disconnection between truth-value operation and semantics leads to counterintuitive conclusions: According to the rules of material implication, as long as the antecedent p is false, regardless of the content of the consequent q, the entire proposition “p→q” is always true. For example, the proposition “if the moon were made of cheese, then the earth would rotate in the opposite direction around the sun” was judged to be a true proposition under material implication. This result completely deviates from human semantic understanding of conditional propositions. In natural language, “if...then...” usually requires a causal, logical or correlative connection between the antecedent and the consequent, while material implication only focuses on the mechanical operation of the truth table, reducing the conditional relationship to a pure combination of truth values.

Secondly, the formal distortion of logical relations makes it difficult for material implication to deal with complex semantic scenarios. Traditional logicians have attempted to make the ambiguity of natural language precise through formal methods, but the excessive simplification of material implication leads to the deviation of logical derivation from daily thinking. For example, in legal reasoning, the validity of the proposition that “if someone intentionally kills a person, then they should bear criminal responsibility” depends on the substantial connection between “intentional homicide” and “criminal responsibility”. However, material implication simplifies it into a truth-value operation, which may lead to the absurd conclusion that “if someone does not intentionally kill a person (the antecedent is false), then any act should bear criminal responsibility (the consequent can be arbitrarily true)”. This distortion not only weakens the practicality of logic as a thinking tool, but also exposes the limitations of formal methods in dealing with semantic richness.

1.2 The innovative path of the logical domain theory

1.2.1 Introduce the closed exhaustive disjunction “y”

In view of the defects of traditional substantive implication, the logic domain theory introduces the closed exhaustive disjunctive symbol “Y” (representing the strong disjunctive relation of “either...or...”) to reconstruct the implication formula:

$$(p\rightarrow q)\longleftrightarrow((\neg p\vee q)\wedge(\neg p\vee p))\longleftrightarrow(\neg p Y(p\wedge q))\longleftrightarrow(\neg p Y q)\rightarrow(\neg p\vee q)$$

The core of this reconstruction lies in emphasizing that “either ¬p or q must be true”, that is, the two form a sub-contrary or contradictory relationship (equivalent to the fact that “p and ¬q are in an opposing or contradictory relationship”). Different from traditional material implication, the logical domain theory requires that the validity of “p→q” must be based on the substantial logical connection between the antecedent and the consequent, rather than a simple combination of truth values. For example, in the proposition that “if it rains (p), then the ground is wet (q)”, “¬pYq” indicates that “either it does not rain, or it rains and the ground is wet”. This statement forcibly establishes a semantic connection between p and q, avoiding the arbitrary implication of irrelevant propositions.

1.2.2 The strict distinction of proposition types

The logical domain theory clearly demarcates the boundary between domain propositions and general propositions:

Domain propositions, represented by “p→q” and “¬pYq”, require a logical connection at the semantic level between the antecedent and the consequent, making ¬p and q form a sub-contrary or contradictory relationship. This proposition reflects the essential cognition of conditional relationships in human thinking, that is, “when p is true, q cannot be false”. For instance, “if a number is divisible by 10 (p), then it is divisible by 5 (q)”, which is “p→q”, and its equivalent form is “¬pYq” (“either the number is not divisible by 10, or it is divisible by 5”), reflecting the necessary connection between p and q.

General propositions, that is, factual propositions, such as “¬p∨q”, only describe the factual state of “not p or q” from the perspective of truth values, without involving semantic connections. For example, “today is not Monday (¬p), or tomorrow is Thursday (q)”. In general propositions, only the combination of truth values is concerned, while the temporal logical relationship between “today” and “tomorrow” is ignored.

The breakthrough of the logical domain theory lies in the introduction of “closed exhaustive disjunction” (symbolized as “Y”, meaning “either...or...”). The closed exhaustive disjunction proposition, that is, the domain proposition (¬p Y q), indicates that either ¬p or q must be true, and both cannot be false simultaneously, thus completely ruling out the possibility of (p∧¬q). This is something that a general proposition (¬p∨q) cannot achieve.

The equivalent form of the proposition (p→q), namely (¬p Y q) (read as “either not p or q”), as an expression of a binary truth function, the validity of its logical semantics is deeply rooted in the formal system of propositional logic. When detached from the established logical domain, factual proposition expressions such as (¬p∨p), (¬ q∨q),

($\neg p \vee q$), etc., due to the lack of constraints of the truth-value operation rules, will lose the foundation of logical deduction. At this point, these expressions can neither be verified through the semantic judgment criteria of well-formed formulas nor participate in effective logical reasoning, thus losing their analytical value at the level of propositional logic. This profoundly reveals that the interpretation and logical analysis of any propositional function must rely on specific logical domain frameworks and rule systems such as ($\neg p \vee p$), ($\neg q \vee q$), etc. Otherwise, it will lead to the dissolution of the logical meaning of the expressions and degenerate into a meaningless accumulation of symbols.

1.2.3 The paradoxical deduction of illegal conversion

Domain proposition can unidirectionally imply a general propositions (that is, when "$p \to q$" is true, "$\neg p \vee q$" must be true), but the two cannot be mutually and equivalently transformed. If this rule is violated, it will lead to illegal logical conversion and give rise to paradoxs.

We know that the valid formula $\neg p \longleftrightarrow ((\neg p \vee q) \wedge (\neg p \vee \neg q))$ holds, and its essence is to restore the logical meaning of "not p" through the conjunction of general propositions. However, if the domain proposition "$p \to q$" is illegally equivalent to the general proposition "$\neg p \vee q$", and the derivation is carried out according to the rules of material implication, the following contradiction will occur:

From "$(p \to q) \longleftrightarrow (\neg p \vee q)$", combined with propositional logic, we can get:
$\neg p \to ((p \to q) \wedge (p \to \neg q))$
In the consequent, since ($p \to \neg q$) is equivalent to ($q \to \neg p$), the formula of the consequent is: $(p \to q) \wedge (q \to \neg p)$, and thus it becomes "$p \to q \to \neg p$", that is "$p \to \neg p$". (If p is true, then not p must be true, which is a contradiction).

Symmetrically, from "$(\neg p \to q) \longleftrightarrow (p \vee q)$", combined with propositional logic, we can get:
$p \to ((\neg p \to q) \wedge (\neg p \to \neg q))$
In the consequent, since ($\neg p \to \neg q$) is equivalent to ($q \to p$), the formula of the consequent is "$(\neg p \to q) \wedge (q \to p)$", and thus it becomes "$\neg p \to q \to p$", that is "$\neg p \to p$". (If $\neg p$ is true, then p must be true, which is a contradiction).

The logical paradox of "$p \longleftrightarrow \neg p$" formed by the above derivation is rooted in the illegal equivalence between domain propositions and general propositions, ignoring the essential differences in semantic relevance between the two.

1.2.4 The unified integration of logical laws

The logical domain theory realizes the organic unity of the law of excluded middle, the law of contradiction, and the law of sufficient reason through the equivalent relation of "$(p \to q) \longleftrightarrow (\neg p \downarrow q) \longleftrightarrow (p \uparrow \neg q)$":
"$\downarrow$" (sub-contrary or contradictory relation) reflects that either $\neg p$ or q must be true, and concretizes the law of excluded middle as the logical constraint between propositions. For example, in the proposition that "if a number is divisible by 10 (p), then it is divisible by 5 (q)", "$\neg p \downarrow q$" ensures that at least one of "a number is not

divisible by 10” and “it is divisible by 5” is true, excluding the invalid proposition where the antecedent is false and the consequent is irrelevant.

“↑” (opposition or contradictory relation) requires that either p or ¬q must be false, reflecting the guarantee of the law contradiction for logical consistency. For example, in the proposition that “if a number is divisible by 10 (p), then it is divisible by 5 (q)”, “p↑¬q” prohibits the contradictory statement that “a number can be divisible by 10 (p), but it can not be divisible by 5”.

A valid conditional proposition “p→q” needs to satisfy both the law of excluded middle, that is, “either ¬p or q must be true”, and the law of non-contradiction, that is, “either p or ¬q must be false”. This integration sublimates the logical laws from scattered rules into an organic system of mutual definition, completely changing the situation where the three laws in traditional logic were independent of each other.

Through the above reconstruction, the logical domain theory not only solves the truth-value paradox of material implication, but also lays a theoretical foundation for logic that is more in line with the essence of human thinking through the deep integration of semantics and logic.

2. The pioneering breakthrough of the conceptual domain theory

2.1 The completion of the conceptual relationship system

2.1.1 The addition of the sub-contrary relationship: the crucial puzzle piece of the logical relationship map

In the study of conceptual relationships, traditional logic has long been confined to four categories of relationships, encompassing six types in total, namely, identity, inclusion (proper inclusion and inclusion), pure intersection (the so-called conceptual intersection relationship), and total difference (opposition and contradiction). The incompleteness of this theoretical system has given rise to numerous difficulties in logical analysis. The logical domain theory has made a breakthrough by introducing the sub-contrary relationship, thus filling this millennial gap. The sub-contrary relationship refers to the logical connection between two concepts, which means that “they cannot both be false, but they can both be true (they cannot both be negated, but they can both be affirmed)”. Its core value lies in the perfection of the rules for logical deduction between concepts.

Based on the principle of logical duality, the logical domain theory clearly states that if S and P are in an opposition relationship, then ¬S and ¬P must be in a sub-contrary relationship. Taking the conceptual domain of “triangle” as an example, “obtuse triangle” and “right triangle” are typically in an opposition relationship, which means that they cannot both be true, but they can both be false (such as in the case of an acute triangle). Its complementary sets, “non-obtuse triangle” and “non-right triangle”, form a sub-contrary relationship: in any classification of triangles, “non-obtuse” and “non-right” cannot both be false (there is no triangle that is neither obtuse nor right), but they can both be true (an acute triangle satisfies both simultaneously). The clarification of this relationship enables logical reasoning to handle negative statements in natural language more precisely, avoiding logical loopholes caused by the lack of such a relationship.

2.1.2 The refined classification of the opposition relationship: a multidimensional deconstruction from the perspective of the logical domain

The definition of the opposition relationship in traditional logic is rather general. Through the conceptual domain polygon model, the logical domain theory subdivides the opposition relationship into three subtypes, significantly improving the precision of logical analysis:

Relative contradictory relationship: The opposition relationship exhibits contradictory characteristics within a specific universe of discourse. For example, “obtuse triangle” and “non-obtuse triangle” form a contradictory relationship (either one or the other) within the conceptual domain of “triangle”, but when placed in the broader domain of “polygon”, they degrade into an opposition relationship (there are other possibilities such as quadrilaterals). This relativity reveals the context-dependence of conceptual relationships and breaks through the cognitive limitation of the absolutization of the contradictory relationship in traditional logic.

Parallel opposition relationship: It is an incompatible relationship between same-level species concepts under the same genus concept. For example, “obtuse triangle”, “right triangle” and “acute triangle” are all sub-concepts of “triangle”, which are mutually exclusive and can both be false (when the object of discussion does not belong to the category of triangles). This type of relationship is the main manifestation of the opposition relationship in traditional logic. However, the logical domain theory makes its logical attributes clearer by clarifying the hierarchical structure.

Non-parallel opposition relationship: It is the incompatibility among concepts across levels. For instance, “triangle” and “square”, “right triangle” and “non-triangle”, etc., belong to geometric concepts at different levels. Although they do not form a opposition relationship at the same level, within the more abstract domain of “plane figure”, the two also have an opposition nature (a figure cannot belong to these two categories simultaneously). This classification expands the applicable scope of the opposition relationship, enabling logical analysis to cover conceptual conflicts in complex semantic scenarios.

2.2 Logical treatment of imaginary concepts and empty concepts

2.2.1 The framework of “conceptual domain w”: a logical revolution breaking through the limitations of reality

In traditional logic, imaginary concepts (such as “unicorn”) and empty concepts (such as “a round square”) are often excluded from logical operations because they lack real-world references, which severely limits the universality of logic. The theory of “conceptual domain W” endows all concepts (whether real or imaginary) with logical legitimacy by introducing the contextual anchoring of the “possible world”. The core of this framework is to liberate logical analysis from the shackles of the real world and instead focus on the logical relationships of concepts within specific semantic fields.

In the “conceptual domain W”, each concept is regarded as a logically real concept. Its existence does not depend on real-world counterparts but derives its meaning through the relational network with other concepts. This setting allows logical rules to function properly in fictional, hypothetical, or abstract contexts, greatly expanding the application boundaries of logic.

2.2.2 Case analysis: the logical rebirth of virtual and real concepts

“Unicorn”: The relationship construction of fictional concepts

In the real world, the “unicorn” is a non-existent virtual concept. However, in the “conceptual domain W of mythical creatures”, it forms a parallel opposition relationship with the “Pegasus” (both belong to mythical creatures but are mutually exclusive), and a relative contradictory relationship or a contradictory relationship with the “non-unicorn” (in the context of mythological narratives, creatures are either one or the other). By clarifying the conceptual domain, the logical domain theory enables propositions such as “the unicorn has one horn” and “the Pegasus has no horn” to obtain valid truth-value judgments, achieving the logical processing of fictional concepts.

“The Round Square”: A clear definition of contradictory concepts

As a typical empty concept, “the round square” cannot exist both in reality and in logic because it violates the law of contradiction. Through the rules of the “conceptual domain W”, the logical domain theory clearly defines it as “an invalid concept that violates the law of contradiction”. This definition avoids the ambiguous treatment of contradictory concepts in traditional logic. Instead of simply excluding them or assigning them meaningless truth values, it confirms their invalidity through logical rules, thus maintaining the consistency of the logical system. In a specific conceptual domain, propositions such as “the round square is not round” and “the round square is not square” can also be true propositions.

This innovative treatment of virtual concepts and empty concepts not only solves the difficult problem of conceptual operations that have long troubled logic, but also drives the shift of logical analysis from “reality mapping” to “thinking construction”. When logical rules can handle all possible concepts without hindrance, logic truly achieves the leap from studying “laws of reality” to exploring “laws of thinking”, providing more powerful analytical tools for fields such as philosophy, linguistics, and artificial intelligence.

By completing the system of conceptual relationships and innovating the rules of conceptual operations, the theory of logical domain has thoroughly transformed the traditional theory of logical concepts. This breakthrough not only fills the theoretical void but also reshapes the relationship between logic and human thinking. Logic is no longer confined to the static classification of real-world objects, instead, it has become a thinking engine capable of dynamically handling diverse semantics and constructing complex conceptual networks.

3. The resolution and prevention of logical paradoxes

3.1 The diagnosis of the essence of paradoxes

3.1.1 The handling defects of self-referential propositions in traditional logic

The existence of logical paradoxes is like the sword of Damocles hanging over the head of logic, challenging the self-consistency of the logical system for a long time. The dilemma of traditional logic regarding paradoxes is centrally reflected in the failure of dealing with self-referential propositions. And the root of this problem lies in the formal defects of material implication. Taking the liar paradox as an example, when the proposition is stated as “this sentence is false”, if it is symbolized as “$p \rightarrow \neg p$” (p represents that “this sentence is false” and $\neg p$ represents that “this sentence is true”), based on the rules of material implication in traditional logic, it will fall into a circular negation where “if p is true, then $\neg p$ must be true; if $\neg p$ is true, then p must be true”. This contradiction stems from the fact that material implication simplifies the conditional relationship into truth-value operations, ignoring the semantic correlation between the antecedent and consequent of the proposition, enabling self-referential propositions to continuously generate logical contradictions through formal rules.

In the framework of traditional logic, the particularity of self-referential propositions is obscured by formal symbols. For instance, in Russell’s paradox (“whether the set consisting of all sets that do not contain itself contain itself”) also leads to the logical collapse of the relationship between “belonging to” and “not belonging to” due to the excessive simplification of material implication. This way of handling not only fails to truly resolve the paradox but also intensifies the generation of contradictions through formal derivation, exposing the vulnerability of traditional logic when dealing with complex semantics.

3.1.2 The judgment criteria of the logical domain theory

The theory of the logical domain fundamentally redefines the determination criteria of valid conditional propositions and provides a brand-new perspective for resolving paradoxes. This theory explicitly requires that a legitimate conditional proposition “$p \rightarrow q$” must satisfy the condition that “p and $\neg q$ form an opposite or contradictory relationship”. This criterion emphasizes the substantial logical connection between the antecedent and the consequent of a proposition, rather than merely the correspondence of truth values. Under this rule, the paradoxical nature of self-referential propositions is directly negated. Because self-reference is essentially the self-negation of the same proposition (for example, in “$p \rightarrow \neg p$”, p and $\neg p$ are actually positive and negative expressions of the same content), they neither form an opposite relationship (an opposite relationship requires the incompatibility of two different concepts) nor a contradictory relationship (a contradictory relationship needs to be based on a clear division of the logical domain), so they do not possess the validity of logical reasoning.

The theory of the logical domain further points out that the emergence of paradoxes stems from the neglect of the semantic relevance of propositions by traditional logic. When the validity of "p→q" relies solely on the truth table, self-referential propositions can take advantage of the loopholes in the formal rules to generate contradictions in the absence of substantial logical connections. By introducing the determination condition of the "opposite or contradictory relationship", the theory of the logical domain sets a semantic threshold for the validity of propositions, cutting off the generation path of paradoxes from the source.

3.2 Examples of paradox resolution

3.2.1 The ineffectiveness proof of the liar paradox "p→¬p"

Taking the liar paradox "This sentence is false" as an example, the theory of the logical domain reveals its invalidity through a three-step analysis:

i. Lack of semantic relevance:
In "p→¬p", both p and ¬p point to the same proposition "This sentence is false", which is essentially a circular expression of self-reference. According to the theory of the logical domain, the antecedent and consequent of a valid conditional proposition need to have independent semantic content. However, here p and ¬p are actually repetitions of the same concept or proposition and cannot form an "opposite or contradictory relationship". For example, if p is defined as "It is raining" and ¬p is defined as "It is not raining", the two form a contradictory relationship because they describe different states. But in the liar paradox, p and ¬p do not provide any substantial distinction, thus they do not meet the basic requirement of a valid proposition.

ii. The logical relationship does not hold:
From the perspective of the equivalent formula of (p→q) in the theory of the logical domain, which is "¬pYq", the liar paradox can be transformed into "¬pY¬p" (either "this sentence is true" or "this sentence is true"). This expression neither conforms to the characteristic of "both cannot be false" of the sub-contrary relationship (it is impossible for two identical propositions to have one true and one false), nor can it form an effective logical deduction. In traditional logic, "p→¬p" is given legitimacy due to the truth-value operation of material implication. However, in the theory of the logical domain, the lack of its internal logical relationship directly leads to its judgment as an invalid proposition.

iii. Cutting off the root of the contradictory cycle:
The logical domain theory negates the inferential value of "p→¬p" by emphasizing semantic relevance. When a self-referential proposition fails to meet the judgment

criteria of valid conditional propositions, the contradictory cycle of “being both true and false” triggered by it loses its logical foundation. This implies that the liar paradox is not an inevitable result of logical rules, but a pseudo-proposition caused by the formal defect of traditional logic.

3.2.2 Redefining the logical boundaries to eliminate the contradictory cycle

The resolution of the liar paradox by the logical domain theory is essentially a redefinition the logical boundaries. Traditional logic, due to its excessive reliance on formalization, includes self-referential propositions within the scope of valid reasoning, resulting in the emergence of logical contradictions. In contrast, the theory of the logical domain, through semantic constraints, clearly demarcates the boundaries of the validity of propositions: any proposition that is self-referential or lacks semantic relevance is not allowed to enter the legitimate field of logical reasoning.

This redefinition of boundaries not only solves the problem of the liar paradox, but also provides a unified solution for other types of logical paradoxes (such as Russell’s paradox and the barber paradox). For example, in Russell’s paradox, the concept of “all sets that do not contain themselves”, due to the contradiction between the relationships of “containing” and “not containing” caused by self-reference, is also judged as an invalid concept under the logical domain theory because of the lack of effective semantic connection. By shifting the focus of logical analysis from formal truth values to semantic essence, the theory of the logical domain has achieved a fundamental transformation in logic from “passively tolerating paradoxes” to “actively preventing paradoxes”, laying a solid foundation for constructing a self-consistent logical system.

To sum up, the logical domain theory has successfully eliminated the threat of logical paradoxes by innovating the criteria for the validity of propositions. This breakthrough not only ended the millennial logical dilemma but also propelled logic back to its essence as the “science of the laws of thought”. It provides reliable logical support for the theoretical development in fields such as philosophy, mathematics, and artificial intelligence.

4. The significance of the paradigm revolution of the logical domain theory

4.1 Methodological shift: from form-priority to semantic-driven cognitive regression

4.1.1 The principle of semantic priority: the foundation of logical rules in natural language

The revolutionary nature of the logical domain theory is first manifested as a fundamental shift in methodology. It has subverted the logical development path since Frege’s principle of “form taking precedence over semantics”, reestablished the cognitive principle of “semantics determining form”, and made logic truly become a “science of thinking” that conforms to the natural thinking of human beings.

Traditional logic pursues the formal simplicity of mathematics and strips the meaning of propositions into pure truth-value symbols (such as “p” and “q”), leading to the disconnection between logical rules and the intuitive thinking in natural language. In contrast, the logical domain theory emphasizes that logical forms must be rooted in semantic associations. For instance, the validity of the conditional proposition “p→q” no longer depends on the mechanical determination of the truth table, but rather on whether “p and ¬q form an opposite or contradictory relationship”. This criterion directly corresponds to the “intuition of meaning association” in human daily thinking. When we say “If we study hard (p), we will get good grades (q)”, the implicit premise is that “studying hard but not getting good grades (p∧¬q)” is unacceptable, which is exactly the semantic anchoring of “p and ¬q cannot both be true” in the logical domain theory.

The profound significance of the principle of semantic priority lies in restoring the essential attribute of logic as a "thinking tool". Traditional logic's formalization overly pursues the self-consistency of the symbolic system, yet forgets that the ultimate goal of logical rules is to serve human cognition. By introducing semantic categories such as “sub-contrary relationship” and “relative contradictory relationship” into the formal system, the logical domain theory brings logical deduction closer to the complex semantic structure of natural language. For example, in the interpretation of legal provisions and the deduction of scientific hypotheses, it can more accurately capture the substantial logical connections between propositions and avoid reasoning fallacies caused by semantic distortion.

4.1.2 Context dependence: the dual-domain definition of proposition validity

The dual-domain framework of the “concept domain W” and the “proposition domain w” proposed by the logical domain theory marks the formal acceptance of context dependence in logic. Traditional logic assumes that propositions have universal validity beyond context. For example, “p→q” is determined solely by truth values. However, the logical domain theory points out that the meaning and truth values of propositions must be examined in specific contexts.

The concept domain W defines the semantic boundaries of concepts. For instance, the contradictory relationship between “obtuse triangle” and “non-obtuse triangle” only holds in the domain of “triangles”, while it is an opposing relationship in the domain of “polygons”.

The proposition domain w restricts the applicable scope of propositions. For example, the proposition “If it rains, then the ground is wet” is valid in the “open-air environment domain”, but does not hold in the “indoor environment domain”.

The theoretical construction of this context dependence breaks the formal myth of the “one-size-fits-all” approach in traditional logic, enabling logical analysis to dynamically adapt to the contextual changes in natural language. For example, in literary criticism, the proposition that “unicorns symbolize purity” has logical legitimacy in the “mythological text domain”, but it is invalid in the “real biological domain” due to the vacuity of the concept. Through the dual-domain framework, the logical domain theory provides precise logical modeling tools for such complex semantic scenarios, promoting logic to return from “symbolic operations divorced from context” to “context-sensitive thinking deductions”.

4.2 The value of interdisciplinary application: from the logical revolution to cognitive empowerment

The innovation of the logical domain theory is not limited to the field of logic itself, , it also provides breakthrough analytical tools for fields such as artificial intelligence, philosophical research, and education through the output of methodology.

4.2.1 Artificial intelligence: the logical cornerstone of contextual reasoning

In the field of artificial intelligence, the construction of knowledge graphs and decision-making in complex scenarios has always been plagued by context dependence and semantic ambiguity. The formal methods of traditional logic find it difficult to handle the polysemy in natural language (for example, “Apple” can refer to both a fruit and a brand) and context switching (such as the different impacts of “rain” in the “traffic domain” and the “agricultural domain”). However, the dual-domain model of the logical domain theory provides a key path to solve this problem:

Context stratification of knowledge graphs: Through the “concept domain W”, hierarchical divisions are made of entity relationships (such as the “mammal domain” and the “marine organism domain”), enabling AI to recognize the validity of “whales are mammals” in the biological domain and avoid cross-domain confusion.

Scene adaptation of decision-making logic: In an autonomous driving system, the validity of the proposition “If heavy rain is detected (p), then drive at a reduced speed (q)” depends on the contextual setting of the “open road domain”. The logical domain theory helps AI systems dynamically adjust reasoning rules in different scenarios (day/night, urban/rural) by clarifying the boundaries of the proposition domain w, achieving decision-makings that conform to human intuition.

The construction of this contextualized reasoning model will drive AI to upgrade from “pattern matching based on data statistics” to “semantic understanding based on logical rules”, laying a logical foundation for the development of general artificial intelligence.

4.2.2 Philosophical research: a new analytical tool for modality and metaphysics

In the field of philosophy, the logical domain theory provides breakthrough solutions to long-standing controversial issues such as modal logic and the ontology of fictional objects:

Intra-domain interpretation of modal propositions: Traditional modal logic's portrayal of "possibility" relies on possible world semantics, but lacks an accurate definition of the concept domain. Through the contextual anchoring of the "concept domain W", the logical domain theory gives logical significance to the proposition "Unicorns may exist" in the "mythical creature domain", while in the "real biological domain", it is only a virtual concept, avoiding cross-domain confusion of modal propositions.

The logical positioning of fictional objects: Regarding metaphysical issues such as "Does a golden mountain exist?" and "Is a round square self-contradictory?", the logical domain theory points out that the logical status of fictional objects should be determined by the concept domain they belong to. "Golden mountain" is a legitimate concept in the "imagination domain" (forming a contrary relationship with "silver mountain"), while "a round square" is invalid in the concept domain of any possible world due to violating the law of contradiction. However, propositions like "A round square is not round" and "A round square is not square" can still be true. This kind of analysis resolves the ontological disputes in traditional philosophy about the "existence" of fictional objects, and transforms the issues into judgments of logical relationships within the concept domain.

In addition, the discovery of the conceptual sub-contrary relationship provides a new tool for semantic analysis in analytic philosophy. For example, when discussing the binary thinking of "either black or white", the logical domain theory reveals that "not black" and "not white" form a sub-contrary relationship in the domain of colors (there is a gray area), thereby providing a logical basis for criticizing extreme thinking.

4.2.3 In the field of education: logical innovation in the cultivation of critical thinking

The core objective of logical education is to cultivate rational thinking ability. However, due to excessive formalization in traditional logical teaching, students often fall into "symbol worship" and overlook the semantic logic in natural language. The logical domain theory innovates logical education through the following approaches:

Breaking the binary opposition thinking: Through the teaching of the sub-contrary relationship, students can understand the logical relationship that "not both false but can both be true (not both negated but can both hold)", for example, in the domain of "youth", there is an intermediate state (ordinary youth) between "non-progressive youth" and "non-backward youth", thus avoiding the simplistic judgment of "either-or".

Strengthening context sensitivity awareness: Through the training of the dual-domain model, students can learn to define the concept domain and the proposition domain first when analyzing propositions. For instance, when analyzing "All swans are white", they can consciously distinguish the different logical truth values between the "domain of real swans" and the "domain of swans in fairy tale".

Logical training that returns to the essence of thinking: Through examples of resolving paradoxes (such as ineffective proof of the liar paradox), students can understand that logical rules are not symbol games divorced from reality, but rather the formal presentation of human thinking laws, thus establishing a logical cognitive view of “semantics first, form as a supplement”.

This educational innovation will contribute to the cultivation of students’ critical thinking. It enables them to conduct rigorous formal reasoning and keenly identify semantic traps in natural language, truly achieving the transformation from logical ability to cognitive ability.

4.3 Historical positioning: the “copernican revolution” in logic

The emergence of the logical domain theory marked a fundamental shift in logic from the “Aristotle-Frege” tradition to the “laws of thinking in natural language” tradition, and its historical significance can be comparable to Copernicus’ “heliocentric theory”.

4.3.1 Transcending traditional formal logic: from symbolic games to the science of thinking

Since Frege mathematized logic, formalization has become the dominant direction of the discipline’s development. Although this approach brought the precision of the symbolic system, it has led to logic gradually becoming a “toy for mathematicians”. The paradoxes of material implication, the incompleteness of conceptual relationships, and the unsolvability of paradoxes are essentially the products of excessive formalization. The revolutionary nature of the logical domain theory lies in its rediscovery of the essential object of logic: logic should not be a mathematical branch that studies the truth values of symbols, but a cognitive science that studies the laws of human thinking.

This transcendence is manifested in three aspects:

Resetting of research objectives: Shifting from pursuing the perfect self-consistency of the symbolic system to explaining and regulating human natural thinking;

Reconstruction of the theoretical foundation: Taking semantic association as the premise of logical form, and not vice versa;

Expansion of the application boundaries: Moving from the narrow circle of mathematical and logical philosophy back to various fields that serve human cognition (such as artificial intelligence, philosophy, and education).

4.3.2 The profound significance of the “copernican revolution”: reconstructing the essential definition of logic

Just as Copernicus subverted the “geocentric theory” and repositioned the earth, the theory of the logical domain has overturned the disciplinary stereotype that “logic is formalization” and endowed logic with new essential connotations:

Logic is the formalization of the laws of thinking, not a mirror image of the real world. The real conceptualization of virtual concepts’ logic demonstrates that the object of logic is the concept domain constructed by thinking, rather than direct mapping of the external world. This provides a theoretical foundation for logic to break free from the limitations of reality and explore possible worlds.

The legitimacy of logical rules stems from human thinking intuition. The discovery of the unity of the three laws and the sub-contrary relationship is essentially a formalized refinement of intuitions such as “either-or” and “not both false” in human natural thinking , providing an empirical basis in cognitive science for logical rules.

Logic serves as a bridge connecting language, thinking, and the world. Through the dual-domain framework, logic has for the first time systematically dealt with the complex relationships among language contexts, thinking rules, and world category, becoming a universal tool for interdisciplinary cognitive research.

From a more macroscopic perspective of intellectual history, the emergence of the logical domain theory marks the awakening of logic from the alienation of “instrumental rationality”. When logical rules once again conform to the essential characteristics of human thinking, they are no longer an intellectual game for a select few, but become a cognitive tool for all people to understand the world and construct knowledge. This return might precisely be the crucial turning point for logic to redefine its own mission after the quagmire of formalization for thousands of years.

The paradigm revolution of the logical domain theory is not only a theoretical breakthrough within the field of logic, but also a significant leap in human beings’ understanding of their own thinking laws. By subverting material implication, completing conceptual relationships, resolving logical paradoxes, and reshaping the essence of logic, it has constructed a new logical system that is “context-sensitive, semantically self-consistent, and open to application”. When the dual-domain framework and the rule of the unity of the three laws of this theory are extended to a broader cognitive field, what we witness is not just the rebirth of logic, but also new possibilities in the entire field of cognitive science. From the contextual reasoning of artificial intelligence to the conceptual analysis in philosophy, from the cultivation of critical thinking in education to the logical construction of possible worlds by humanity, the theory of the logical domain is weaving a new cognitive network that connects thinking and existence.

The history of logic is the history of humanity’s continuous pursuit of the thinking’s clarity. The emergence of the logical domain theory marks that we have finally broken through the cocoon of formalization and started to freely navigate in the infinite domain of thinking. This might not be the end, but a new starting point. At this starting point, logic truly becomes the “science of thinking”, and humanity’s exploration of the essence of its own cognition thus embarks on a broader journey.

5. Conclusion summary of theoretical value and future prospects

The logical domain theory has accomplished a systematic paradigmatic innovation of traditional logic with four core breakthroughs: subverting material implication, completing conceptual relationships, unifying logical laws, and resolving logical paradoxes.

This theory not only puts an end to the truth-value paradoxes caused by material implication, the incompleteness of the conceptual relationship theory, and the insoluble dilemmas of logical paradoxes, but also reshapes the essence of logic as the "science of thinking" through the dual-track mode of "semantic anchoring + logical deduction", enabling logical rules to return from a formal symbolic game divorced from semantics to the essential laws of human natural thinking. Its theoretical value combines historical breakthroughs and practical extensibility:

Internally within the field of logic, it fills the conceptual gaps in the history of both Chinese and Western logic, and constructs a self-consistent semantic-logic unified system.

In external applications, it provides brand-new tools for the contextual reasoning of artificial intelligence, the modal analysis in philosophy, and the cultivation of critical thinking in education. This marks a significant leap in human cognition of thinking laws, from the formal surface level to the semantic essence.

Looking ahead, the "dual-domain" analytical framework and the "unifying the three laws" rule of the logical domain theory are expected to be further extended to fields such as cognitive science, linguistics, and decision science, facilitating the construction of context-sensitive logical models in interdisciplinary research. As the theory is applied implemented in more complex semantic scenarios, it may lead logic to evolve from a "breaker" for long-standing dilemmas to a "thinking engine" that reshapes the human cognitive paradigm, providing an eternal theoretical foundation for exploring the essence of intelligence, the logic of language, and the construction of possible worlds.

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